

The following problem accompanies the book, Method of Weighted Residuals and Variational Principles, by Bruce A. Finlayson, a SIAM Classic reprinted in 2014. The original version was printed by Academic Press in 1972. See [www.ChemEComp.com](http://www.ChemEComp.com)/MWR. Order the book from the Society of Industrial and Applied Mathematics, [www.SIAM.org](http://www.SIAM.org). The problems and solutions refer to equations and references in that book.

**Problem 3.** Solve the same problem as in Problem 2 but using the second order finite difference method. The error bounds are not applicable to this approximation, but compare the results with those in Problem 2. The problem is:

$$\frac{d^2 y}{dx^2} + \lambda y = 0, y(0) = y(1) = 0$$

**Part a.** Evaluate the variational integral using

$$\left. \frac{dy}{dx} \right|_i = \frac{y_{i+1} - y_i}{\Delta x}$$

Differentiate the variational integral with respect to  $y_j$ .

**Part b.** Solve the equations in part a to get the approximate eigenvalues. Are they upper bounds? Compare the errors to those in Problem 2.