

The following problem accompanies the book, Method of Weighted Residuals and Variational Principles, by Bruce A. Finlayson, a SIAM Classic reprinted in 2014. The original version was printed by Academic Press in 1972. See www.ChemEComp.com/MWR. Order the book from the Society of Industrial and Applied Mathematics, www.SIAM.org. The problems and solutions refer to equations and references in that book.

Problem 4. Solve the same problem as in Problem 2 but using the orthogonal collocation method. The error bounds are not applicable because the variational integral is not being minimized, but compare the results with those in Problem 2. The problem is:

$$\frac{d^2 y}{dx^2} + \lambda y = 0, y(0) = y(1) = 0$$

The orthogonal collocation method is described in detail in §5.1 and the roots are given in Table 5.4. A program that will generate the matrices in Table 5.5 is planar.m.

Part a. Verify that the MATLAB function planar.m gives the matrices in Table 5.5.

Part b. Then compute the eigenvalues for $n = 3-7$ (n = total number of points, 1-5 interior points). (remember that the value of the solution at the first and last point is zero, so that only the interior part of the matrix is used. Alternatively, you can replace the first row of the full matrix B_{ij} with zeroes except for the first element, $i=1$, ($=1$), and do the same for the last row except for the last element, $i=n$ ($=1$). Are the eigenvalues above the exact eigenvalues? Compare the error versus the number of terms.