

The following problem accompanies the book, Method of Weighted Residuals and Variational Principles, by Bruce A. Finlayson, a SIAM Classic reprinted in 2014. The original version was printed by Academic Press in 1972. See www.ChemEComp.com/MWR. Order the book from the Society of Industrial and Applied Mathematics, www.SIAM.org. The problems and solutions refer to equations and references in that book.

Problem 5 Solve the following problem, Eq. (11.86).

$$u_{xx} - u_t - g(x,t)u = f(x,t), \quad u(x,0) = u(0,t) = u(\pi,t) = f(0,0) = f(\pi,0) = 0$$

when

$$g(x,t) = 1, \quad f(x,t) = -\sum_{j=1}^5 \frac{\sin jx}{10^{j-1}}.$$

Use the expansion function, Eq. (11.87).

$$u_n(x,t) = \sum_{k=1}^n C_{nk}(t) \sin kx, \quad C_{nk}(0) = 0$$

Part a. Solve for $n = 1, 2, \dots, 6$.

Part b. Calculate the error bound, Eq. (11.89) for each n .