The following problem accompanies the book, Method of Weighted Residuals and Variational Principles, by Bruce A. Finlayson, a SIAM Classic reprinted in 2014. The original version was printed by Academic Press in 1972. See www.ChemEComp.com/MWR. Order the book from the Society of Industrial and Applied Mathematics, www.SIAM.org. The problems and solutions refer to equations and references in that book.

Problem 4. Part a. Using the MATLAB program planar.m we get, for example,

B = [4, -8, 4;   4, -8, 4;   4, -8, 4]

as expected (see page 106). The other matrices are correct, too, for A and the next approximation.

Part b. Since the value at the two end points is zero, the matrix to define the eigenvalues is

$$\sum_{i=1}^{n} B_{ji} y_i + \lambda y_j = 0$$

MATLAB solves in the form

$$\sum_{i=1}^{n} B_{ji} y_i = \lambda y_j$$

so that the eigenvalues are the negative of the ones we want. This gives the matrix B as

B = [1-λ, 0 0;   4, -8- λ, 4; 0. 0. 1- λ]

Thus, the eigenvalues are the roots to

$$-(1-\lambda)^2(8+\lambda)=0$$

The first approximation thus gives the eigenvalue -8, or for our problem 8. The second approximation gives

B = [-24 -λ, 12; 12, –24 – -λ]

or $\left(\lambda+24\right)^2=144,\quad \lambda+24=\pm\sqrt{144},\quad \lambda=-12,-36$. Thus we get 12 and 36 for the first two eigenvalues.

A MATLAB code that evaluates the matrices and gives the eigenvalues is:

```
%eigenvalue using orthogonal collocation
clear all
global xoc woc  aoc boc qinv
n=7
kon=0 % 0 to print the matrics
planar(n,kon)
for i=2:n-1
   for j=2:n-1
      A(i-1,j-1)=boc(i,j);
   end
end
A
lambda=-eig(A)
```

Results are as follows.

n	3	4	5	6	7	exact
$\lambda 1$	8	12	9.7733	9.8751	9.8695	9.8696044
$\lambda 2$		36	60	37.8046	39.7649	39.47842
$\lambda 3$			98.227	170.125	82.058	88.82644

Errors are:

n	3	4	5	6	7	exact
$\lambda 1$	-1.87	2.13	-0.0963	0.00550	0.000200	9.8696044
$\lambda 2$		-3.48	20.52	-1.67	0.286	39.47842
$\lambda 3$			9.40	81.3	-6.77	88.82644

The errors oscillate in sign and are somewhat large. However, as the approximation improves, the errors decrease dramatically. This is a feature of the orthogonal collocation method.